Basic Reserve Allocation Math for Fully Insured Programs

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Background

A granular database of incurred and paid claims and earned premium is quite valuable. Incurred and paid loss ratios can be easily drawn from the database in myriad levels of aggregation, limited only by the structural granularity of the identifying characteristics for each cell of data. The most recent months of this claims data are frequently considered functionally incomplete; there has simply not been enough elapsed time for all, or even most, of the incurred claims to have become processed and paid. As a consequence, actuaries typically produce analysis on rating and distribution variables by ignoring the last few months of the data, by estimating and adding a reserve component to each of their separate analyses, or by developing analysis on a fully paid claims basis rather than an incurred and paid claims basis.

A robust alternative to these approaches has the actuary allocating best estimate reserves into the database, with an eye on minimizing introduced bias. This approach allows the actuary to produce analysis that uses all of the information, including critical emerging information from the latest few months.

Notation

m	$=$	$accrual\ month$
n	$=$	$duration\ month$
CF_m	$=$	$experience\ completion\ factor\ for\ month\ m$
CF'_m	$=$	$allocation\ completion\ factor\ for\ month\ m$
DF_n	$=$	$experience\ duration\ factor\ for\ duration\ n$
DF'_n	$=$	$allocation\ duration\ factor\ for\ duration\ n$
$C_{m,n}$	$=$	$incurred\ and\ paid\ claims\ for\ month\ m\ and\ duration\ n$
$P_{m,n}$	$=$	$estimated\ earned\ premium\ for\ month\ m\ and\ duration\ n$
$IC_{m,n}$	$=$	$estimated\ incurred\ claims\ for\ month\ m\ and\ duration\ n$
$R_{m,n}$	$=$	$estimated\ claims\ reserve\ for\ month\ m\ and\ duration\ n$

Note that premium is normally used for as a proxy for exposure since it is periodically adjusted on a case by case basis by underwriting processes to reflect all available information at the time of the adjustment. As an alternative, enrollment or expected claims may be used in place of premium, with appropriate alternative adjustments to the duration factor construct.

Understanding the rationale for the reserve allocation formula

When allocating a claims reserve into a granular database, it is important to do so in a way that preserves the underlying characteristics of the data, especially the most recent data. Many actuaries will throw out the most recent few months' data on the grounds that it is immature, losing the information that lies within. Careful attention to detail when allocating will allow for the use of even the most recent month of data.

A good estimate for incurred claims when the data cell is nearly complete:

$$IC_{m,n} = \frac{C_{m,n}}{CF_m}$$

A good estimate for incurred claims when the data cell is not very complete:

$$IC_{m,n} = C_{m,n} + (1 - CF_m)P_{m,n}DF_n$$

The duration factor is an expected loss ratio construct, applied to premium to provide an expected claims figure. Duration, expressed as months since issue, is the most critical indicator of expected claims ratios in small employer insurance portfolios, especially during the initial case year. Two other indicators can be the natural calendar month and the size at issue of the group. These additional indicators may also be considered in this duration factor form, though the derivation and smoothing processing becomes a bit more complex.

A better estimate for incurred claims, blending the first two estimates by completeness:

$$IC_{m,n} = (CF_m)\frac{C_{m,n}}{CF_m} + (1 - CF_m)\big(C_{m,n} + (1 - CF_m)P_{m,n}DF_n\big)$$

$$IC_{m,n} = C_{m,n} + (1 - CF_m)C_{m,n} + (1 - CF_m)^2 P_{m,n}DF_n$$

$$IC_{m,n} = (2 - CF_m)C_{m,n} + (1 - CF_m)^2 P_{m,n}DF_n$$

So the estimate for claims reserves for a cell of data is generally:

$$R_{m,n} = (1 - CF_m)C_{m,n} + (1 - CF_m)^2 P_{m,n}DF_n$$

The algorithm

$Step\ 1$ $-$ $Estimate\ the\ claims\ reserves\ by\ past\ accrual\ months\ and\ in\ total\ using\ Bornhuetter-Ferguson\ methods\ performed\ on\ completion\ ratio\ analysis\ of\ claim\ lag\ triangles\ producing$:

$$R_m\ and\ \ CF_m\ for\ all\ m$$

$Step\ 2$ $-$ $Determine\ the\ n\ duration\ factors$:

$$DF_n = \frac{\sum_m (2 - CF_m) C_{m,n}}{\sum_m (2 - CF_m) CF_m P_{m,n}}$$

$Step\ 3$ $-$ $Smooth\ the\ duration\ factors\ to\ create\ allocation\ duration\ factors$.

$Step\ 4$ $-$ $Determine\ allocation\ completion\ factors\ so\ that\ they\ reproduce\ original\ reserves\ by\ accrual\ month$:

$$Ratio_m = \frac{\sum_n C_{m,n}}{\sum_n P_{m,n} DF'_n}$$

$$CF'_m = 1 - \frac{1}{2} Ratio_m \left(-1 + \sqrt{1 + \frac{4\left(\frac{1}{CF_m} - 1\right)}{Ratio_m}} \right)$$

$Step\ 5$ $-$ $Perform\ the\ allocation\ of\ reserves$:

$$R_{m,n} = (1 - CF'_m) C_{m,n} + (1 - CF'_m)^2 P_{m,n} DF'_n$$

<u>Step 1 notes</u>

Estimate the claims reserves by past accrual months and in total using Bornhuetter-Ferguson methods performed on completion ratio analysis of claim lag triangles. Notation is as follows:

$$C_m = \sum_n C_{m,n}$$

$$IC_m = \frac{C_m}{CF_m}$$

$$R_m = IC_m - C_m = \left(\frac{1}{CF_m} - 1\right) C_m$$

$$R = \sum_m R_m = \sum_m IC_m - \sum_m C_m = \sum_m \left(\frac{1}{CF_m} - 1\right) C_m$$

<u>Step 2 notes</u>

Determine the n duration factors.

Duration factors are defined as incurred loss ratios by monthly duration:

$$DF_n = \frac{IC_n}{P_n} = \frac{\sum_m IC_{m,n}}{\sum_m P_{m,n}}$$

Substitute the expression for incurred claims from the allocation rationale:

$$DF_n = \frac{\sum_m \left((2 - CF_m)C_{m,n} + (1 - CF_m)^2 P_{m,n} DF_n\right)}{\sum_m P_{m,n}}$$

$$DF_n = \frac{\sum_m (2 - CF_m)C_{m,n} + \sum_m (1 - CF_m)^2 P_{m,n} DF_n}{\sum_m P_{m,n}}$$

$$DF_n = \frac{\sum_m (2 - CF_m)C_{m,n} + DF_n \sum_m (1 - CF_m)^2 P_{m,n}}{\sum_m P_{m,n}}$$

$$DF_n \sum_m P_{m,n} = \sum_m (2 - CF_m) C_{m,n} + DF_n \sum_m (1 - CF_m)^2 P_{m,n}$$

$$DF_n \sum_m P_{m,n} - DF_n \sum_m (1 - CF_m)^2 P_{m,n} = \sum_m (2 - CF_m) C_{m,n}$$

$$DF_n \sum_m (1 - (1 - CF_m)^2) P_{m,n} = \sum_m (2 - CF_m) C_{m,n}$$

$$DF_n = \frac{\sum_m (2 - CF_m) C_{m,n}}{\sum_m (1 - (1 - CF_m)^2) P_{m,n}}$$

$$DF_n = \frac{\sum_m (2 - CF_m) C_{m,n}}{\sum_m (2 - CF_m) CF_m P_{m,n}}$$

These are the monthly duration factors that are reflected by the data in the database with the completion factors that determine the already estimated claims reserve.

<u>Step 3 notes</u>

Smooth the duration factors to create allocation duration factors.

The duration factors are determined solely from the cash data and the original completion factors. These duration factors ought to be smoothed since they represent estimates of emerging claims where the months are immature. Whittaker-Henderson techniques work well, but many other methods will do nicely.

$$DF' = smoothed\ DF$$

If the duration factors are more complex, reflecting plan year deductibles for example, then the smoothing process will be more complex.

Step 4 notes

Determine allocation completion factors so that they reproduce original reserves by accrual month.

Since the claims reserves by accrual months are to be reproduced, solve for the allocation completion factors directly:

$$R_m = \sum_n R_{m,n} = \left(\frac{1}{CF_m} - 1\right)\sum_n C_{m,n} = \sum_n (1 - CF'_m) C_{m,n} + \sum_n (1 - CF'_m)^2 P_{m,n} DF'_n$$

$$\left(\frac{1}{CF_m} - 1\right)\sum_n C_{m,n} = (1 - CF'_m)\sum_n C_{m,n} + (1 - CF'_m)^2 \sum_n P_{m,n} DF'_n$$

$$0 = (1 - CF'_m)^2 \sum_n P_{m,n} DF'_n + (1 - CF'_m)\sum_n C_{m,n} - \left(\frac{1}{CF_m} - 1\right)\sum_n C_{m,n}$$

Use the quadratic formula to produce an expression for allocation completion factor complements:

$$1 - CF'_m = \frac{-\sum_n C_{m,n} + \sqrt{\left(\sum_n C_{m,n}\right)^2 + 4\left(\frac{1}{CF_m} - 1\right)\sum_n C_{m,n} \sum_n P_{m,n} DF'_n}}{2\sum_n P_{m,n} DF'_n}$$

Solve for the allocation completion factors:

$$CF'_m = 1 + \frac{\sum_n C_{m,n}}{2\sum_n P_{m,n} DF'_n} - \frac{\sum_n C_{m,n}}{2\sum_n P_{m,n} DF'_n}\sqrt{1 + 4\left(\frac{1}{CF_m} - 1\right)\frac{\sum_n P_{m,n} DF'_n}{\sum_n C_{m,n}}}$$

$$CF'_m = 1 - \frac{1}{2}\frac{\sum_n C_{m,n}}{\sum_n P_{m,n} DF'_n}\left(-1 + \sqrt{1 + 4\left(\frac{1}{CF_m} - 1\right)\frac{\sum_n P_{m,n} DF'_n}{\sum_n C_{m,n}}}\right)$$

Create a ratio measure for each month, akin to a paid actual-to-expected ratio:

$$Ratio_m = \frac{\sum_n C_{m,n}}{\sum_n P_{m,n} DF'_n}$$

Then the final formula for allocation completion factors:

$$CF'_m = 1 - \frac{1}{2} Ratio_m \left(-1 + \sqrt{1 + \frac{4\left(\frac{1}{CF_m} - 1\right)}{Ratio_m}}\right)$$

If the ratio measure equals the original completion factor, which is its natural estimator, then the allocation factor reproduces the original completion factor:

$$CF'_m = 1 - \frac{1}{2}\,CF_m\left(-1 + \sqrt{1 + \frac{4\left(\frac{1}{CF_m} - 1\right)}{CF_m}}\right)$$

$$CF'_m = 1 + \frac{1}{2}\,CF_m - \frac{1}{2}\,CF_m\sqrt{1 + \frac{4(1 - CF_m)}{(CF_m)^2}}$$

$$CF'_m = 1 + \frac{1}{2}\,CF_m - \frac{1}{2}\sqrt{(CF_m)^2 - 4\,CF_m + 4}$$

$$CF'_m = 1 + \frac{1}{2}\,CF_m - \frac{1}{2}(2 - CF_m)$$

$$CF'_m = CF_m$$

Step 5 notes

Perform the allocation of reserves.

$$R_{m,n} = (1 - CF'_m)C_{m,n} + (1 - CF'_m)^2 P_{m,n} DF'_n$$