Actuarial Strategies and Tactics

Probabilistic Reserve Estimation for Aggregate-Only Stop Loss Programs

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Background

Aggregate-only stop loss programs, especially those programs with designed low aggregate factors, require special handling. Normal stop loss reserving methods, those involving development triangles, are appropriate for stop loss programs where the vast majority of the risk is specific stop loss risk. Alongside this triangle development method, aggregate reserving is normally performed on an open claim and inspection basis. These methods will not work effectively on aggregate-only programs where the probability of an aggregate claim is high; there are simply too many aggregate claims to adequately and unbiasedly perform reserves by inspection. A systematic and algorithmic approach is required.

A granular database of monthly incurred and paid claims, monthly funding, and monthly earned aggregate premium is quite valuable. Such a database will allow for estimation and allocation of first dollar reserves, enabling case-by-case projection of the remaining aggregate accumulation year. These deterministic case-by-case projections of claims can be used as point estimate inputs for an appropriate probabilistic model designed to produce case-by-case aggregate claims ratios.

Accrual Definitions

Most insurance companies consider the accrual date of medical insurance claims the date of service. A few still consider the accrual date to be the accident date or the date of onset for an illness; however, their insurance plans tend to be something other than comprehensive major medical plans. With an aggregate insurance program there is no date of service upon which the insurer can rely for reporting purposes. Instead, an aggregate claim is allocable on an accrual basis to the underlying plan year.

This distinction is critical. It means that the reserves for the aggregate program are not simply a portion of all of the plan reserves. It means that the expected aggregate claim for each case must be estimated, with a portion of it allocated to already earned aggregate premium, and the balance allocated to future earned premium. Total aggregate reserves are the sum of all case aggregate reserves.

Reserves for the underlying plan are a hybrid of the two approaches. The plan must reflect its experience and a service date basis, but it will take a reserve credit equivalent to the liability that the stop loss carrier calculates for itself.

<u>Notation for Key Financial Measures</u>

Since the probabilistic reserving process is performed case-year by case-year, the subscript notation is simplified to a single indicator. Financial measures are delineated by a subscript indicating the duration month of the case-year. Previous two dimensional duration and month nomenclature is not needed. An appropriate translation of the data into this more simplified structure is required.

The case-year is partitioned into two partitions, months that have already occurred, designated “past” with superscript “P” and months that are projected, designated “future” with superscript “F.” Financial figures without such subscripts will generally be indicating full case-year amounts.

Items in bold in the table below are estimated items, while the non-bolded items are accumulations from the working database after plan reserves have been allocated.

$\underline{Measure}$	$\underline{Month\ k}$	$\underline{Past}$	$\underline{Future}$	$\underline{Full\ Year}$
$Enrollment$	N_k	N^P	$\boldsymbol{N^F}$	$\boldsymbol{N}$
$Plan\ Leveled\ Expected\ Gross\ Claims$	E_k	E^P	$\boldsymbol{E^F}$	$\boldsymbol{E}$
$Funding\ (= Attachment\ Factors)$	F_k	F^P	$\boldsymbol{F^F}$	$\boldsymbol{F}$
$Aggregate\ Premium$	P_k	P^P	$\boldsymbol{P^F}$	$\boldsymbol{P}$
$Plan\ Gross\ Cash\ Claims$	C_k	C^P		
$Plan\ Claim\ Reserves$	R_k	R^P		
$Plan\ Gross\ Incurred\ Claims$	$\boldsymbol{IC_k}$	$\boldsymbol{IC^P}$	$\boldsymbol{IC^F}$	$\boldsymbol{IC}$
$Aggregate\ Cash\ Claims$	ACC_k	ACC^P		
$Plan\ Net\ Cash\ Claims$	$\boldsymbol{PCC_k}$	$\boldsymbol{PCC^P}$		
$Aggregate\ Accrual\ Claims$	$\boldsymbol{AAC_k}$	$\boldsymbol{AAC^P}$	$\boldsymbol{AAC^F}$	$\boldsymbol{AAC}$
$Plan\ Net\ Accrual\ Claims$	$\boldsymbol{PAC_k}$	$\boldsymbol{PAC^P}$	$\boldsymbol{PAC^F}$	$\boldsymbol{PAC}$
$Aggregate\ Claims\ Reserve$	$\boldsymbol{AR_k}$	$\boldsymbol{AR^P}$		

In general, the following relationship holds for these financial measures:

$$\sum_{1}^{12} measure_k = measure^P + measure^F = measure$$

In particular, for these past values:

$$IC_k = C_k + R_k \qquad IC^P = C^P + R^P$$
$$PCC_k = C_k - ACC_k \quad PCC^P = C^P + ACC^P$$

Math Constructs

A number of intermediate values and parameters are required. The parameters are **bolded**, the rest of the items are produced by calculation, except for the first item. Notation for these are as follows:

$Number\ of\ months\ in\ past\ partition\ of\ the\ plan\ year$	L
$Duration\ factors\ from\ plan\ reserve\ allocation\ process$	$\boldsymbol{DF'_k}$
$Duration\ deterioration\ factor\ between\ plan\ year\ partitions$	β
$Fully\ credible\ projected\ experience\ for\ the\ future\ partition$	$IC^F_{Z=1}$
$Non\ credible\ projected\ experiencefor\ the\ future\ partition$	$IC^F_{Z=0}$
$Credibility\ factor\ for\ blending\ projected\ experience$	Z
$Credibility\ measure\ reflecting\ size\ and\ completeness$	M
$Credibility\ constant\ parameter\ (if\ M = K, then\ Z = .5)$	$\boldsymbol{K}$
$Probability\ density\ function\ for\ actual\ to\ expected\ claims$	$\boldsymbol{f_X}(\boldsymbol{x})$
$Adjusted\ attachment\ point\ for\ density\ integration$	α
$Portfolio\ performance\ adjustment\ factor$	$\boldsymbol{\delta}$

Step 1 – Project Future Plan Year Measures

Use a simple flat line approach to produce future values for enrollment, leveled expected claims, employer funding, and aggregate premium:

$$N^F = (12 - L)N_L \quad N = N^P + N^F$$
$$E^F = (12 - L)E_L \quad E = E^P + E^F$$
$$F^F = (12 - L)F_L \quad F = F^P + F^F$$
$$P^F = (12 - L)P_L \quad P = P^P + P^F$$

Step 2 – Estimate Future Plan Year Claims

On a case by case basis, the future incurred claims are estimated deterministically, that is, as a single point estimate. A fully credible projection of incurred claims is credibility blended with an estimate based on case-year pricing.

An adjustment for intra-year deterioration in the projection is reflected by manipulating duration factors. These factors have trend already reflected in their structure:

$$\beta = \frac{\sum_{L+1}^{12} DF_k' E_k}{\sum_1^{12} DF_k' E_k}$$

A portfolio performance adjustment factor can be used to adjust the zero credibility experience, especially when the actuary deems that the portfolio is performing significantly above or below expectations. The factor is entirely up to the actuary's discretion, and may be left at simply one; however, a good starting point to consider is:

$$\delta = \frac{\sum_{portfolio} IC_k}{\sum_{portfolio} DF_k' E_k}$$

The zero credibility estimate of the projected plan year incurred claims is the *a priori* part of the total leveled expected claims adjusted by the effect of deterioration, then adjusted by the portfolio performance factor:

$$IC_{Z=0}^F = E^F \frac{\sum_{L+1}^{12} DF_k' \frac{E_k}{E^F}}{\sum_1^{12} DF_k' \frac{E_k}{E}} \delta = E\beta\delta$$

The fully credible projected plan year incurred claims are the past incurred claims adjusted by the change in exposure, then adjusted by the deterioration factor:

$$IC_{Z=1}^F = IC^P \left(\frac{E^F}{E^P}\right) \frac{\sum_{L+1}^{12} DF_k' \frac{E_k}{E^F}}{\sum_1^{L} DF_k' \frac{E_k}{E^P}} = IC^P \frac{\beta}{1-\beta}$$

The credibility factor for each case-year is produced by first calculating a measure based on size and completeness of experience:

$$M = \frac{C^P N^P}{IC^P}$$

Credibility factors follow by using a credibility constant established for the entire program. This parameter is the key variable to adjust when the actuary is fine-tuning the reserving process:

$$Z = \frac{M}{M+K}$$

Projected future incurred claims are then a credibility weighting of the two estimates:

$$IC^F = (Z)IC^F_{Z=1} + (1-Z)IC^F_{Z=0}$$

$$IC^F = (Z)IC^P \frac{\beta}{1-\beta} + (1-Z)E\beta\delta$$

This measure is the key point estimate that will allow for the generation of a parameter in the probabilistic calculation for the case-year aggregate claim. An alternative formulation:

$$IC^F = \delta\beta E + \ Z\left(IC^P \frac{\beta}{1-\beta} - \delta\beta E\right)$$

By altering the credibility constant and the portfolio performance adjustment factor, the actuary can produce results over a range of values. The sensitivity of the results to movements of these two variables provide additional insight for the actuary's assessment of the experience.

<u>Step 3 – Estimate the Aggregate Accrued Claim</u>

The aggregate accrued claim is calculated by using a special probability density function, just as the net premium at time of issue is calculated. A family of density functions is inherent in the aggregate stop loss pricing structures. Each one is standardized so that its mean is 1.0, reflecting its appropriateness for simulating outcomes that are ratios to expected claims.

At issue, a case-year's net premium is calculated using a density function based on size, underwriting load, and duration year. When estimating the aggregate claim while the case-year is progressing, the size used is recalculated to reflect a larger variance for the density function:

$$Density\ function\ size\ lookup = \frac{\frac{R^P}{IC^P} N^P + N^F}{12}$$

This resulting density function can be used to integrate for the expected aggregate claim, given the financial development of the case-year to date.

The random variable that corresponds with the new density function is a claims ratio, and it has an expected value of 1. To integrate for the expected aggregate claim factor, a measure for an adjusted attachment point is set up:

$$\alpha = \frac{F - C^P}{R^P + IC^F}$$

If this measure is less than zero, it means that the estimated aggregate accrued claim is deterministically available from point estimates:

$$AAC = (R^P + IC^F)(1 - \alpha) = R^P + IC^F + C^P - F$$

If this measure is greater than zero, then the following integration provides the estimated aggregate accrued claim:

$$AAC = (R^P + IC^F) \int_{\alpha}^{\infty} (x - \alpha) f_X(x) dx$$

$$AAC = (R^P + IC^F) \left(\int_{\alpha}^{\infty} x f_X(x) dx - \alpha \int_{\alpha}^{\infty} f_X(x) dx \right)$$

This value is an unbiased estimate for the case-year aggregate accrued claim, and it is available for allocation across the case-year.

<u>Step 4 – Allocate the Aggregate Accrued Claim and Produce Aggregate Reserves</u>

The aggregate accrued claim is allocated by earned aggregate premium.

$$AAC_k = \left(\frac{P_k}{P^P + P^F} \right) AAC$$

$$AAC^P = \left(\frac{P^P}{P^P + P^F} \right) AAC$$

$$AAC^F = \left(\frac{P^F}{P^P + P^F} \right) AAC$$

The aggregate reserve for the case year is the difference between the past accrued aggregate claim and the past cash aggregate claims:

$$AR_k = AAC_k - ACC_k$$

$$AR^P = AAC^P - ACC^P$$

The past plan accrued claims is the past plan cash claims and the allocated reserve for past service dates' unpaid claims, but credited with the past allocated aggregate accrued claim:

$$PAC_k = IC_k - ACC_k$$

$$PAC^P = IC^P - AAC^P$$

$$PAC^F = IC^F - AAC^F$$

$$PAC = IC - AAC$$