Reserve Allocation Math for Aggregate-Only Stop Loss Programs

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Background

Aggregate-only stop loss programs, especially those programs with designed low aggregate factors, require special handling. Normal stop loss reserving methods, those involving development triangles, are appropriate for stop loss programs where the vast majority of the risk is specific stop loss risk. Alongside this triangle development method, aggregate reserving is normally performed on an open claim and inspection basis. These methods will not work effectively on aggregate-only programs where the probability of an aggregate claim is high; there are simply too many aggregate claims to adequately and unbiasedly perform reserves by inspection. A systematic and algorithmic approach is required.

A granular database of monthly incurred and paid claims, monthly funding, and monthly earned aggregate premium is quite valuable. Such a database will allow for estimation and allocation of first dollar reserves, enabling case-by-case projection of the remaining aggregate accumulation year. This note lays out the framework for allocating full plan claims reserves, a necessary first prerequisite for the stochastic estimation process that follows.

Notation

$m = accrual\ month$
$n = duration\ month$
$P_{m,n} = estimated\ earned\ aggregate\ premium\ for\ month\ m\ and\ duration\ n$
$F_{m,n} = employer\ funding\ for\ month\ m\ and\ duration\ n\ (also\ the\ attachment\ point)$
$C_{m,n} = incurred\ and\ paid\ claims\ for\ month\ m\ and\ duration\ n$
$s = expected\ surplus\ factor\ relative\ to\ expected\ claims\ (varies\ by\ case)$
$g = underwriting\ gain\ relative\ to\ expected\ claims$
$e = expense\ margin\ on\ aggregate\ premium$
$CF_m = experience\ completion\ factor\ for\ month\ m$
$CF'_m = allocation\ completion\ factor\ for\ month\ m$
$DF_n = experience\ duration\ factor\ for\ duration\ n$
$DF'_n = allocation\ duration\ factor\ for\ duration\ n$
$E_{m,n} = estimated\ expected\ claims\ for\ month\ m\ and\ duration\ n\ (leveled\ exposure)$
$IC_{m,n} = estimated\ incurred\ claims\ for\ month\ m\ and\ duration\ n$
$R_{m,n} = estimated\ claims\ reserve\ for\ month\ m\ and\ duration\ n$

The Rationale for the Reserve Allocation Formula

When allocating a claims reserve into a granular database, it is important to do so in a way that preserves the underlying characteristics of the data, especially the most recent data. Many actuaries will throw out the most recent few months' data on the grounds that it is immature, losing the information that lies within. Careful attention to detail when allocating will allow for the use of even the most recent month of data.

A good estimate for incurred claims when the data cell is nearly complete:

$$IC_{m,n} = \frac{C_{m,n}}{CF_m}$$

A good estimate for incurred claims when the data cell is not very complete:

$$IC_{m,n} = C_{m,n} + (1 - CF_m)E_{m,n}DF_n$$

The duration factor is an actual-to-expected ratio construct, applied to leveled expected claims to provide a monthly expected claims figure based on the emerging experience in the database. Duration, expressed as months since issue, is the most critical indicator of expected claims ratios in small employer plan experience, especially during the initial case year. Two other indicators can be the natural calendar month and the size at issue of the group. These additional indicators may also be considered in this duration factor form, though the derivation and smoothing processing becomes a bit more complex.

A better estimate for incurred claims, blending the first two estimates by completeness:

$$IC_{m,n} = (CF_m)\frac{C_{m,n}}{CF_m} + (1 - CF_m)\big(C_{m,n} + (1 - CF_m)E_{m,n}DF_n\big)$$

$$IC_{m,n} = C_{m,n} + (1 - CF_m)C_{m,n} + (1 - CF_m)^2 E_{m,n}DF_n$$

$$IC_{m,n} = (2 - CF_m)C_{m,n} + (1 - CF_m)^2 E_{m,n}DF_n$$

So the estimate for claims reserves for a cell of data is generally:

$$R_{m,n} = (1 - CF_m)C_{m,n} + (1 - CF_m)^2 E_{m,n}DF_n$$

The Algorithm

$Step\ 1 \quad - \quad Calculate\ leveled\ expected\ claims$:

$$E_{m,n} = \frac{P_{m,n}(1-e) + F_{m,n}}{1+g+s}$$

$Step\ 2 \quad - \quad Estimate\ the\ claims\ reserves\ by\ past\ accrual\ months\ and\ in\ total\ using\ Bornhuetter - Ferguson\ methods\ performed\ on\ completion\ ratio\ analysis\ of\ claim\ lag\ triangles\ producing$:

$$R_m\ and\ \ CF_m\ for\ all\ m$$

$Step\ 3 \quad - \quad Determine\ the\ n\ duration\ factors$:

$$DF_n = \frac{\sum_m (2 - CF_m) C_{m,n}}{\sum_m (2 - CF_m) CF_m E_{m,n}}$$

$Step\ 4 \quad - \quad Piecewise\ smooth\ the\ duration\ factors\ to\ create\ allocation\ duration\ factors.$

$Step\ 5 \quad - \quad Determine\ allocation\ completion\ factors\ so\ that\ they\ reproduce\ original\ reserves\ by\ accrual\ month$:

$$Ratio_m = \frac{\sum_n C_{m,n}}{\sum_n E_{m,n} DF'_n}$$

$$CF'_m = 1 - \frac{1}{2} Ratio_m \left(-1 + \sqrt{1 + \frac{4\left(\frac{1}{CF_m} - 1\right)}{Ratio_m}} \right)$$

$Step\ 6 \quad - \quad Perform\ the\ allocation\ of\ reserves$:

$$R_{m,n} = (1 - CF'_m) C_{m,n} + (1 - CF'_m)^2 E_{m,n} DF'_n$$

Step 1 – Calculate Expected Claims

Leveled expected claims is not a transaction that is captured; its values must be estimated from the transaction data and a few assumptions. Leveled expected claims is necessary as a measure of exposure in much of the reserving process.

If the original expected surplus factor from the original quote is available it is best to use it in the derivation of expected claims. If it is not available, the actuary makes a careful and unbiased estimate. This factor will generally be a figure between 0.05 and 0.09.

Leveled expected claims by month is aggregate premium less expenses and gain, plus funding less expected surplus:

$$E_{m,n} = P_{m,n} - eP_{m,n} - gE_{m,n} + F_{m,n} - sE_{m,n}$$

$$E_{m,n} = \frac{P_{m,n}(1-e) + F_{m,n}}{1+g+s}$$

Step 2 – Estimate Plan Claims Reserves

Estimate the claims reserves by past accrual months and in total using Bornhuetter-Ferguson methods performed on completion ratio analysis of claim lag triangles. Use the leveled expected claim as the exposure measure in the estimation process. Notation is as follows:

$$C_m = \sum_n C_{m,n}$$

$$IC_m = \frac{C_m}{CF_m}$$

$$R_m = IC_m - C_m = \left(\frac{1}{CF_m} - 1\right) C_m$$

$$R = \sum_m R_m = \sum_m IC_m - \sum_m C_m = \sum_m \left(\frac{1}{CF_m} - 1\right) C_m$$

<u>Step 3 – Determine Duration Factors</u>

Duration factors are defined as actual-to-expected ratios by monthly duration:

$$DF_n = \frac{IC_n}{E_n} = \frac{\sum_m IC_{m,n}}{\sum_m E_{m,n}}$$

Substitute the expression for incurred claims from the allocation rationale:

$$DF_n = \frac{\sum_m \left((2 - CF_m)C_{m,n} + (1 - CF_m)^2 E_{m,n} DF_n\right)}{\sum_m E_{m,n}}$$

$$DF_n = \frac{\sum_m (2 - CF_m)C_{m,n} + \sum_m (1 - CF_m)^2 E_{m,n} DF_n}{\sum_m E_{m,n}}$$

$$DF_n = \frac{\sum_m (2 - CF_m)C_{m,n} + DF_n \sum_m (1 - CF_m)^2 E_{m,n}}{\sum_m E_{m,n}}$$

$$DF_n \sum_m E_{m,n} = \sum_m (2 - CF_m)C_{m,n} + DF_n \sum_m (1 - CF_m)^2 E_{m,n}$$

$$DF_n \sum_m E_{m,n} - DF_n \sum_m (1 - CF_m)^2 E_{m,n} = \sum_m (2 - CF_m)C_{m,n}$$

$$DF_n \sum_m (1 - (1 - CF_m)^2) E_{m,n} = \sum_m (2 - CF_m)C_{m,n}$$

$$DF_n = \frac{\sum_m (2 - CF_m)C_{m,n}}{\sum_m (1 - (1 - CF_m)^2) E_{m,n}}$$

$$DF_n = \frac{\sum_m (2 - CF_m)C_{m,n}}{\sum_m (2 - CF_m) CF_m E_{m,n}}$$

These are the monthly duration factors that are reflected by the data in the database with the completion factors that determine the already estimated claims reserve.

Step 4 – Smooth the Duration Factors

Smooth the duration factors to create allocation duration factors.

The duration factors are determined solely from the cash data and the original completion factors. These duration factors ought to be smoothed and extrapolated piecewise by plan year, reflecting the renewal selection effects that naturally occur. Whittaker-Henderson techniques work well, but many other methods will do nicely

$$DF' = smoothed\ DF$$

If the duration factors are more complex, reflecting plan year deductibles or group size for example, then the smoothing process will be more complex.

Step 5 – Create Allocation Completion Factors

Determine allocation completion factors so that they reproduce original reserves by accrual month.

Since the claims reserves by accrual months are to be reproduced, solve for the allocation completion factors directly:

$$R_m = \sum_n R_{m,n} = \left(\frac{1}{CF_m} - 1\right)\sum_n C_{m,n} = \sum_n (1 - CF'_m) C_{m,n} + \sum_n (1 - CF'_m)^2 E_{m,n} DF'_n$$

$$\left(\frac{1}{CF_m} - 1\right)\sum_n C_{m,n} = (1 - CF'_m)\sum_n C_{m,n} + (1 - CF'_m)^2 \sum_n E_{m,n} DF'_n$$

$$0 = (1 - CF'_m)^2 \sum_n E_{m,n} DF'_n + (1 - CF'_m)\sum_n C_{m,n} - \left(\frac{1}{CF_m} - 1\right)\sum_n C_{m,n}$$

Use the quadratic formula to produce an expression for allocation completion factor complements:

$$1 - CF'_m = \frac{-\sum_n C_{m,n} + \sqrt{\left(\sum_n C_{m,n}\right)^2 + 4\left(\frac{1}{CF_m} - 1\right)\sum_n C_{m,n} \sum_n E_{m,n} DF'_n}}{2\sum_n E_{m,n} DF'_n}$$

Solve for the allocation completion factors:

$$CF'_m = 1 + \frac{\sum_n C_{m,n}}{2\sum_n E_{m,n} DF'_n} - \frac{\sum_n C_{m,n}}{2\sum_n E_{m,n} DF'_n}\sqrt{1 + 4\left(\frac{1}{CF_m} - 1\right)\frac{\sum_n E_{m,n} DF'_n}{\sum_n C_{m,n}}}$$

$$CF'_m = 1 - \frac{1}{2}\frac{\sum_n C_{m,n}}{\sum_n E_{m,n} DF'_n}\left(-1 + \sqrt{1 + 4\left(\frac{1}{CF_m} - 1\right)\frac{\sum_n E_{m,n} DF'_n}{\sum_n C_{m,n}}}\right)$$

Create a ratio measure for each month, akin to a paid actual-to-expected ratio:

$$Ratio_m = \frac{\sum_n C_{m,n}}{\sum_n E_{m,n} DF'_n}$$

Then the final formula for allocation completion factors:

$$CF'_m = 1 - \frac{1}{2} Ratio_m \left(-1 + \sqrt{1 + \frac{4\left(\frac{1}{CF_m} - 1\right)}{Ratio_m}}\right)$$

If the ratio measure equals the original completion factor, which is its natural estimator, then the allocation factor reproduces the original completion factor:

$$CF'_m = 1 - \frac{1}{2}\ CF_m \left(-1 + \sqrt{1 + \frac{4\left(\frac{1}{CF_m} - 1\right)}{CF_m}}\right)$$

$$CF'_m = 1 + \frac{1}{2}\ CF_m - \frac{1}{2}\ CF_m \sqrt{1 + \frac{4(1 - \ CF_m)}{(\ CF_m)^2}}$$

$$CF'_m = 1 + \frac{1}{2}\ CF_m - \frac{1}{2}\sqrt{(\ CF_m)^2 - 4\ CF_m + 4}$$

$$CF'_m = 1 + \frac{1}{2}\ CF_m - \frac{1}{2}(2 - \ CF_m)$$

$$CF'_m = \ CF_m$$

Step 6 – Allocate Reserves

Perform the allocation of reserves using:

$$R_{m,n} = (1 - CF'_m)C_{m,n} + (1 - CF'_m)^2 E_{m,n} DF'_n$$